Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 63255

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Second Semester

Civil Engineering

MA 1151 — MATHEMATICS — II

(Common to all branches)

(Regulations 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Laplace transform of $\frac{\sin at}{t}$.

2. Find the inverse Laplace transform of $\frac{s+1}{s^2-2s}$.

3. The temperature at a point (x, y, z) in a space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at $(1,1,2)$ desires to fly in such a direction that it will get warm as soon as possible. In what direction should it fly?

4. A vector field $\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$. Find div $\vec{F}$ and Curl $\vec{F}$.

5. Prove that $w = z^2$ is analytic.

6. Find the fixed points of the transformation $w = \frac{6z-9}{z}$.

7. Evaluate : $\int_0^2 \int_0^x (x+y)\, dxdy$.

8. Evaluate $\iint e^{-x-y}\,dxdy$ over R, where R is the region in the first quadrant in which $x+y\le 1$.

9. Identify and classify the singularity of $f(z)=\dfrac{\sin z}{z}$.

10. Find the residue of $f(z)=\dfrac{z+1}{(z-1)(z-2)}$ at $z=2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the Laplace transform of $\dfrac{1-e^t}{t}$. (8)

(ii) Find the Laplace transform of $f(t)=\begin{cases} t, & 0<t<a \\ 2a-t, & a<t<2a \end{cases}$ with $f(t+2a)=f(t)$. (8)

Or

(b) (i) Using convolution theorem find $L^{-1}\left\{\dfrac{1}{s^2(s^2+1)}\right\}$. (8)

(ii) Solve $y''+7y'+10y=4e^{-3t}$, $y(0)=0$, $y'(0)=-1$ using Laplace transform. (8)

12. (a) (i) Prove that $\vec{F}=(y^2\cos x+z^3)\vec{i}+(2y\sin x-4)\vec{j}+(3xz^2+2)\vec{k}$ is irrotational and find its scalar potential. (6)

(ii) Verify Stoke's theorem for $\vec{F}=x^2\vec{i}+xy\vec{j}$ in the square region in the xy-plane bounded by the lines $x=0$, $y=0$, $x=a$, $y=a$. (10)

Or

(b) (i) Prove that $\nabla^2\left(r^n\vec{r}\right)=n(n+3)r^{n-2}$. (6)

(ii) Verify Gauss divergence theorem for $\vec{F}=\left(x^2-yz\right)\vec{i}+\left(y^2-zx\right)\vec{j}+\left(z^2-xy\right)\vec{k}$ taken over the rectangular parallelopiped $0\le x\le a$, $0\le y\le b$, $0\le z\le c$. (10)

13. (a) (i) Let $f(z) = u + iv$ be an analytic function. If $u - v = e^x(\cos y - \sin y)$, then find f. (8)

(ii) If $f(z)$ is an analytic function of z, then prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2$. (8)

Or

(b) (i) Determine the region of w-plane into which the triangle formed by $x = 1$, $y = 1$, $x + y = 1$ is mapped under the transformation $w = z^2$. (8)

(ii) Find the bilinear transformation which maps the points $z = 1, i, -1$ onto the points $w = i, 0, -i$. Hence find the image of $|z| < 1$. (8)

14. (a) (i) Evaluate through change of variables the double integral $\iint_R (x + y)^3 e^{x-y}\, dA$, where R is the square with vertices $(1,0)$, $(2,1)$, $(1,2)$ and $(0,1)$ using the transformation $u = x + y$ and $v = x - y$. (8)

(ii) Find, by triple integral, the volume of the tetrahedron bounded by coordinate planes and the plane $x + y + z = 1$. (8)

Or

(b) (i) Find, by double integration, the area lying inside the circle $r = a \sin\theta$ and outside the cardiod $r = a(1 - \cos\theta)$. (8)

(ii) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$. (8)

15. (a) (i) Using Cauchy's integral formula, evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-2)(z-3)} dz$, where C is the circle $|z| = 4$. (8)

(ii) Evaluate, using contour integration, $\int_0^{2\pi} \frac{d\theta}{1 - 2p\cos\theta + p^2}$, $0 < p < 1$. (8)

Or

(b) (i) Find the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the regions $1 < |z+1| < 2$ and $|z+1| > 2$. (8)

(ii) Evaluate $\int_{-\infty}^{\infty} \frac{x^2\, dx}{(x^2 + a^2)(x^2 + b^2)}$, using contour integration where $a > b > 0$. (8)
